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NONLINEAR SYSTEMS

Constructing Lyapunov Functions in Partial Stability Problems

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Abstract—This paper considers systems of nonlinear ordinary differential equations with a continuous right-hand side. For such systems, stability analysis of the trivial equilibrium with respect to part of the variables is performed in the context of the direct Lyapunov method. Lyapunov functions are constructed by choosing an appropriate pair of a main Lyapunov function and an auxiliary function: the latter function is used to correct the domain of the state space of the system where the former function is introduced and analyzed. Illustrative examples are provided in which such pairs are constructed for some nonlinear systems of two and three differential equations.

Keywords: system of nonlinear ordinary differential equations, partial stability, Lyapunov function method, construction of Lyapunov functions

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1. INTRODUCTION

Along with stability problems with respect to all variables, modern control theory also addresses more general ones with respect to part of the variables (partial stability), going back to the classical works of A.M. Lyapunov [1] and V.V. Rumyantsev [2, 3]. These problems naturally arise in applications and are considered for models with various mathematical descriptions, e.g., continuous- and discrete-time dynamics, the hereditarity effect (delays) and random disturbances, etc., see the reviews in [3–5]. New stability problems have also been investigated: input-output stability [6, 7] and finite-time output stability [8–10].

As in the case of stability with respect to all variables, the main rigorous method for analyzing partial stability is the Lyapunov function method with an appropriate modification. It has been successfully used, e.g., in the stability analysis of complex mechanical systems [3, 11]. However, despite the proven reversibility of several theorems underlying this method and its further development, the most urgent challenge still is to construct Lyapunov functions both for general classes of systems and for particular systems.

In many cases, stability with respect to all variables "in the small" is analyzed based on the linear approximation. In contrast, partial stability analysis even in this situation is *nonlocal* due to "uncontrolled" variables and depends, to a much greater extent, on the structure and parameters of the systems. Such an analysis can be successfully performed by using a suitable Lyapunov function.

This paper considers a possible approach to finding Lyapunov functions in the partial stability problem of the trivial equilibrium for a system of nonlinear differential equations with a continuous right-hand side. It is based on a transition to the analysis of "*extended*" partial stability, i.e.,

not only with respect to an initially specified part of the variables but also with respect to some additional variables (functions of the state variables of the system of differential equations under study). This transition neither changes nor refines the essential meaning of the original partial stability problem but has another goal: to *narrow* the domain of the system's state space where a suitable Lyapunov function is searched, thereby facilitating its choice to ensure required properties. Naturally, according to the meaning of the problem being solved, extended partial stability is an intermediate concept between stability with respect to an initially specified part of the variables and stability with respect to all variables (full stability).

This approach is formalized by finding, along with a main Lyapunov function, one more (generally speaking, vector-valued) auxiliary function that defines a set of additional variables and considering both as a *pair* (bundle) of functions for solving the original partial stability problem. In this case, the auxiliary function is used to correct the domain of the system's state space where the main Lyapunov function is introduced and its properties are analyzed. The requirements imposed on the main Lyapunov function are *weaker* than in the stability analysis with respect to an initially specified part of the variables.

Structural forms of nonlinear systems of two and three differential equations are identified for which the indicated pairs can be constructed and the partial stability domains can be found in the parameter space. The extended partial stability analysis of these structural forms reduces to the stability analysis with respect to all variables based on the linear approximation of equilibria of special auxiliary systems of differential equations obtained from the original system by corresponding transformations of the state variables. These structural forms are used below as a nonlinear approximation for partial stability analysis of systems with polynomial nonlinearities.

2. PROBLEM STATEMENT

Consider a nonlinear system of ordinary differential equations of the form

$$\begin{gathered}\mathbf{x}' = \mathbf{X}(t,\mathbf{x}), \quad \mathbf{X}(t,\mathbf{0}) \equiv \mathbf{0},\\ \mathbf{x} = (x_1,\dots,x_n) = (y_1,\dots,y_m,z_1,\dots,z_p) = (\mathbf{y},\mathbf{z}), \quad m+p=n,\end{gathered} \tag{1}$$

with two groups of variables in the state vector $\mathbf{x}$, namely, 1) the variables $\mathbf{y}$, with respect to which the stability of the equilibrium $\mathbf{x}=\mathbf{0}$ is studied, and 2) the other (uncontrolled) variables $\mathbf{z}$ [3, 4, 12]. From this point onwards, $\|\mathbf{x}\|$ indicates the Euclidean norm of a vector $\mathbf{x}$.

We introduce conventional assumptions from the theory of partial stability ($\mathbf{y}$-stability) [3, 4], i.e., the continuity of the vector function $\mathbf{X}$ in the domain

$$t \geqslant 0, \quad \|\mathbf{y}\| \leqslant h, \quad \|\mathbf{z}\| < \infty, \tag{2}$$

as well as the uniqueness and $\mathbf{z}$-continuability of solutions of system (1). (In other words, each solution $\mathbf{x}(t)$ is defined for all $t \geqslant 0$ such that $\|\mathbf{y}(t)\| \leqslant h$.) Let $\mathbf{x}(t) = \mathbf{x}(t;t_0,\mathbf{x}_0)$ denote the solution of system (1) with an initial condition $\mathbf{x}_0 = \mathbf{x}(t_0;t_0,\mathbf{x}_0)$.

Definitions. The equilibrium $\mathbf{x}=\mathbf{0}$ of system (1) is said to be:

1) $\mathbf{y}$-*stable* if, for any $\varepsilon > 0$ and $t_0 \geqslant 0$, there exists a number $\delta(\varepsilon,t_0) > 0$ such that $\|\mathbf{x}_0\| < \delta$ implies $\|\mathbf{y}(t;t_0,\mathbf{x}_0)\| < \varepsilon$ for all $t \geqslant t_0$;

2) *uniformly* $\mathbf{y}$-*stable* if $\delta = \delta(\varepsilon)$ in definition 1);

3) *asymptotically* $\mathbf{y}$-*stable* if it is $\mathbf{y}$-stable in the sense of definition 1) and there exists $\Delta(t_0) > 0$ such that each solution $\mathbf{x}(t;t_0,\mathbf{x}_0)$ of system (1) with $\|\mathbf{x}_0\| < \Delta$ satisfies the limit relation

$$\|\mathbf{y}(t;t_0,\mathbf{x}_0)\| \to 0, \quad t \to \infty; \tag{3}$$

here, the inequality $\|\mathbf{x}_0\| < \Delta$ determines the domain of $\mathbf{y}$-*attraction* of the equilibrium $\mathbf{x}=\mathbf{0}$ of system (1) for the initial time instant t_0;

4) *uniformly asymptotically* $\mathbf{y}$*-stable* if it is uniformly $\mathbf{y}$-stable in the sense of definition 2), Δ is independent of t_0, and the limit relation (3) holds uniformly in $\{t_0, \mathbf{x}_0\}$ from the domain $t_0 \geqslant 0$, $\|\mathbf{x}_0\| < \Delta$.

For analyzing the $\mathbf{y}$-stability of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1) in the context of the direct Lyapunov method, we consider the following functions:

1) a scalar function $V(t, \mathbf{x})$, $V(t, \mathbf{0}) \equiv 0$, continuously differentiable in the domain (2), as well as its derivative V' along the trajectories of system (1), as a Lyapunov function;

2) a vector function $\mathbf{w}(\mathbf{x})$, $\mathbf{w}(\mathbf{0}) \equiv \mathbf{0}$, continuous in the domain (2), used to estimate from above the Lyapunov function and its derivative;

3) $a(r)$, $b(r)$, and $c(r)$, standard continuous functions used in stability theory, monotonically increasing for $r \in [0, h]$, such that $a(0) = b(0) = c(0) = 0$ (Hahn functions [3]).

According to Rumyantsev [2], if system (1) in the domain

$$t \geqslant 0, \quad \|\mathbf{y}\| < h_1 \leqslant h, \quad \|\mathbf{z}\| < \infty \tag{4}$$

with a sufficiently small number $h_1 > 0$ has a function V such that

$$V(t, \mathbf{x}) \geqslant a(\|\mathbf{y}\|), \quad V'(t, \mathbf{x}) \leqslant 0, \tag{5}$$

then the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1) is $\mathbf{y}$-stable.

With conditions (5) replaced by

$$a(\|\mathbf{y}\|) \leqslant V(t, \mathbf{x}) \leqslant b(\|\mathbf{u}(\mathbf{x})\|), \quad V'(t, \mathbf{x}) \leqslant -c(\|\mathbf{u}(\mathbf{x})\|),$$
$$\mathbf{u}(\mathbf{x}) = [\mathbf{y}, \mathbf{w}(\mathbf{x})],$$

the equilibrium $\mathbf{x} = \mathbf{0}$ becomes uniformly asymptotically $\mathbf{y}$-stable [3, 4, 12].

In stability analysis with respect to all variables, the construction of a Lyapunov function not only settles the stability issue itself but also gives some estimates for the domain of attraction. They are obtained by approximating the domain of attraction by sets lying inside level surfaces of Lyapunov functions.

Several analytical and numerical methods for constructing Lyapunov functions and estimating the domains of attraction were described, e.g., in the monographs [13–16], the reviews [17–20], and the works [21–27]; systems with a discontinuous right-hand side were considered in [28]. We also mention the *divergent* method for stability analysis [29], an alternative to the Lyapunov function method.

The stability problem with respect to part of the variables ($\mathbf{y}$-stability) has certain peculiarities. Even the $\mathbf{y}$-stability analysis of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1) "in the small" leads to a nonlocal problem of investigating the properties of the Lyapunov function in the cylindrical domain (4) of the state space. Moreover, $\mathbf{y}$-stability analysis "in the small" (with respect to $\mathbf{y}_0$) may allow for large initial perturbations of the uncontrolled variables $\mathbf{z}$ (with respect to $\mathbf{z}_0$). Finding Lyapunov functions for such situations is a topical and little-studied problem in the theory of partial stability.

3. THE PROPOSED APPROACH TO FINDING LYAPUNOV FUNCTIONS

In the general case, when investigating the $\mathbf{y}$-stability problem of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1) by the direct Lyapunov method, Lyapunov functions depend on t, $\mathbf{y}$, and, moreover, on the uncontrolled variables $\mathbf{z}$.

Therefore, the assumption $\|\mathbf{z}\| < \infty$, which defines the domain (4) to select and analyze the Lyapunov function, may turn out to be overly restrictive, significantly complicating the verification of the sign definiteness properties of the functions and their derivatives along the trajectories of system (1).

Additional possibilities appear in the transition from the original $\mathbf{y}$-stability problem to the analysis of extended partial stability, i.e., $(\mathbf{y}, \boldsymbol{\mu})$-stability of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1). In this case, the domain (4) is replaced by a narrower one in the state space:

$$t \geqslant 0, \quad \|\mathbf{y}\| + \|\boldsymbol{\mu}(\mathbf{x})\| < h_1 \leqslant h, \quad \|\mathbf{z}\| < \infty, \tag{6}$$

where $\boldsymbol{\mu}(\mathbf{x})$ is a function continuous in the domain (2) such that $\boldsymbol{\mu}(\mathbf{0}) \equiv \mathbf{0}$.

This approach is motivated by the following fact: in the $\mathbf{y}$-stability analysis of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1), extended $(\mathbf{y}, \boldsymbol{\mu})$-stability always holds for some combinations of the state variables defining the function $\boldsymbol{\mu}(\mathbf{x})$. Specifying a particular function $\boldsymbol{\mu}$, however, requires a separate analysis; therefore, this approach is formalized by finding, along with the *main Lyapunov function* $V(t, \mathbf{x})$, one more (generally speaking, vector-valued) *auxiliary* function $\boldsymbol{\mu}(\mathbf{x})$ for the corresponding correction of the domain (6), where the main Lyapunov function is introduced. These functions are treated as a *pair* $\{V(t, \mathbf{x}), \boldsymbol{\mu}(\mathbf{x})\}$ for solving the $\mathbf{y}$-stability problem of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1).

With such a pair, during the solution procedure one can use Lyapunov functions $V(t, \mathbf{x})$ whose derivatives along the trajectories of system (1) may be *indefinite* (of alternating sign) in the domain (4). (And even the functions themselves may have this property.)

Theorem 1. *Suppose that for system* (1)*, it is possible to specify a pair* $\{V(t, \mathbf{x}), \boldsymbol{\mu}(\mathbf{x})\}$ *such that*

$$V(t, \mathbf{x}) \geqslant a(\|\mathbf{y}\| + \|\boldsymbol{\mu}(\mathbf{x})\|), \quad V'(t, \mathbf{x}) \leqslant 0 \tag{7}$$

for sufficiently small $h_1 > 0$ *in the domain* (6)*.*

Then the equilibrium $\mathbf{x} = \mathbf{0}$ *of system* (1) *is* $\mathbf{y}$*-stable.*

If conditions (7) *are replaced by*

$$a(\|\mathbf{y}\| + \|\boldsymbol{\mu}(\mathbf{x})\|) \leqslant V(t, \mathbf{x}) \leqslant b(\|\mathbf{u}(\mathbf{x})\|), \quad V'(t, \mathbf{x}) \leqslant -c(\|\mathbf{u}(\mathbf{x})\|), \tag{8}$$

then the equilibrium $\mathbf{x} = \mathbf{0}$ *is uniformly asymptotically* $\mathbf{y}$*-stable.*

This result is proved via the scheme indicated in [2–4, 12].

Within the proposed approach to the $\mathbf{y}$-stability analysis of the equilibrium $\mathbf{x} = \mathbf{0}$ of system (1), as a candidate for the main function $V(t, \mathbf{x})$ in the pair, it is natural to take the quadratic form

$$V(t, \mathbf{x}) = \mathbf{y} A_1(t) \mathbf{y}^{\mathrm{T}} + \boldsymbol{\mu}(\mathbf{x}) A_2(t) \boldsymbol{\mu}^{\mathrm{T}}(\mathbf{x}) \tag{9}$$

of the variables $\mathbf{y}$ and $\boldsymbol{\mu}$ with appropriately chosen elements of the matrix functions A_1 and A_2 of the corresponding dimensions, where the superscript T means transpose. (In particular, these elements may be independent of t.) Alternatively, a higher-order matrix form of the variables $\mathbf{y}$ and $\boldsymbol{\mu}$ can be adopted: $V(t, \mathbf{x}) \equiv V^*(t, \mathbf{y}, \boldsymbol{\mu}(\mathbf{x}))$.

This pair design scheme is constructively implemented if system (1) can be represented as

$$\mathbf{y}' = \mathbf{Y}^*(t, \mathbf{y}, \boldsymbol{\mu}) + \mathbf{Y}^{**}(t, \mathbf{x}), \quad \boldsymbol{\mu}' = \mathbf{M}(t, \mathbf{y}, \boldsymbol{\mu}) + \mathbf{M}^*(t, \mathbf{x}), \quad \mathbf{z}' = \mathbf{Z}(t, \mathbf{x});$$

and the issue of the $(\mathbf{y}, \boldsymbol{\mu})$-stability of the trivial equilibrium of the resulting system is settled by investigating stability with respect to *all* variables of the trivial equilibrium of the first (linear or nonlinear) approximation system

$$\mathbf{y}' = \mathbf{Y}^*(t, \mathbf{y}, \boldsymbol{\mu}), \quad \boldsymbol{\mu}' = \mathbf{M}(t, \mathbf{y}, \boldsymbol{\mu}). \tag{10}$$

Then Lyapunov functions of the form (9) can be constructed as Lyapunov functions for system (10) by using well-known (analytical or numerical) stability analysis methods with respect to *all* variables.

Note that several partial stability conditions based on the linear and nonlinear approximation were given in [2–5, 30, 31].

Another way of choosing a candidate for the main function $V(t,\mathbf{x})$ involves the first integrals of system (1). Starting from the works of N.G. Chetaev [32], the construction of Lyapunov functions in the form of a bundle of first integrals has been used in the stability analysis of mechanical systems with respect to all or part of the variables [3, 5]. In the $\mathbf{y}$-stability problem of the equilibrium $\mathbf{x}=\mathbf{0}$ of system (1), the introduction of an additional function $\boldsymbol{\mu}(\mathbf{x})$ allows simplifying, in a number of cases, the Lyapunov function structure via the representation

$$V(t,\mathbf{x})=V^*(t,\mathbf{y},\boldsymbol{\mu}(\mathbf{x}))+W(t,\mathbf{x}), \tag{11}$$

where $V^*(t,\mathbf{y},\boldsymbol{\mu}(\mathbf{x}))$ is a sign definite function (in $\mathbf{y}$ and $\boldsymbol{\mu}$) in the domain (6), and the function $W(t,\mathbf{x})$ satisfies the condition $W(t,\mathbf{x})\geqslant 0$ in the domain (6).

For linear systems with constant coefficients, an algorithm for choosing the function $\boldsymbol{\mu}(\mathbf{x})$ and constructing Lyapunov functions in the form (9) was considered in [12].

Remark 1. In pairs, a scalar main Lyapunov function $V(t,\mathbf{x})$ may be replaced by a vector-valued main function $\mathbf{V}=[V_1(t,\mathbf{x}),\dots,V_k(t,\mathbf{x})]$, and such a function is analyzed in the domain (6) by the comparison method [33].

Remark 2. The Lyapunov function–feedback pair (stabilizing pair) was introduced for stabilization of control systems [34]; the Lyapunov function–vector field pair [35] and a correcting auxiliary function to construct a Lyapunov function directly from the right-hand side of the system were applied for stability analysis with respect to all variables [36].

The idea of constructing the pair $\{V(t,\mathbf{x}),\boldsymbol{\mu}(\mathbf{x})\}$, which can be regarded as a solution of the Lyapunov function problem in partial stability analysis, was not stated in general form. However, the $\mathbf{y}$-stability conditions (7), to be verified in the domain (6), were indicated and discussed in [4, 5, 12]; and the uniform asymptotic $\mathbf{y}$-stability conditions (8) were announced in [5, 12]. In this connection, we note that within the proposed approach, the pair can also be chosen under other conditions on the Lyapunov function in the domain (6), different from (8).

Remark 3. For a chosen auxiliary function $\boldsymbol{\mu}(\mathbf{x})$, verification of conditions (7) and (8) in the domain (6) gets simplified if the Lyapunov function and the nonlinearities in system (1) are *polynomial* and the solutions of system (1) starting in a small neighborhood of the equilibrium $\mathbf{x}=\mathbf{0}$ are known to be bounded in the variable $\mathbf{z}$. If the auxiliary system (10) can be *identified* from the original one (1), then it suffices to verify the sign definiteness of the constructed Lyapunov function $V(t,\mathbf{y},\boldsymbol{\mu}(\mathbf{x}))$ and its derivative along the auxiliary system with respect to *all* variables.

Example 1. The motion of a holonomic mechanical system in a potential force field is governed by the Lagrange equations of the second kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial x_i'}\right)-\frac{\partial T}{\partial x_i}=-\frac{\partial \Pi}{\partial x_i},$$

where T is the kinetic energy, Π is the potential energy, and x_i are generalized coordinates.

Let the imposed links have no explicit dependence on time t, and let the system admit the trivial equilibrium $\mathbf{x}=\mathbf{x}'=\mathbf{0}$. We divide the components of the generalized coordinate vector $\mathbf{x}$ into two groups, $\mathbf{x}=(\mathbf{y},\mathbf{z})$, and assume the existence of an auxiliary function $\boldsymbol{\mu}(\mathbf{x})$ such that, in the domain

$$\|\mathbf{y}\|+\|\boldsymbol{\mu}(\mathbf{x})\|+\|\mathbf{x}'\|<h_1\leqslant h,\quad \|\mathbf{z}\|<\infty, \tag{12}$$

the following conditions are satisfied:

1) T is positive definite with respect to the generalized velocities.

2) Π does not depend explicitly on time t and is positive definite with respect to $\mathbf{y}$ and, moreover, with respect to the additional variables $\boldsymbol{\mu}(\mathbf{x})$, i.e., $\Pi(\mathbf{x})\geqslant a(\|\mathbf{y}\|+\|\boldsymbol{\mu}(\mathbf{x})\|)$.

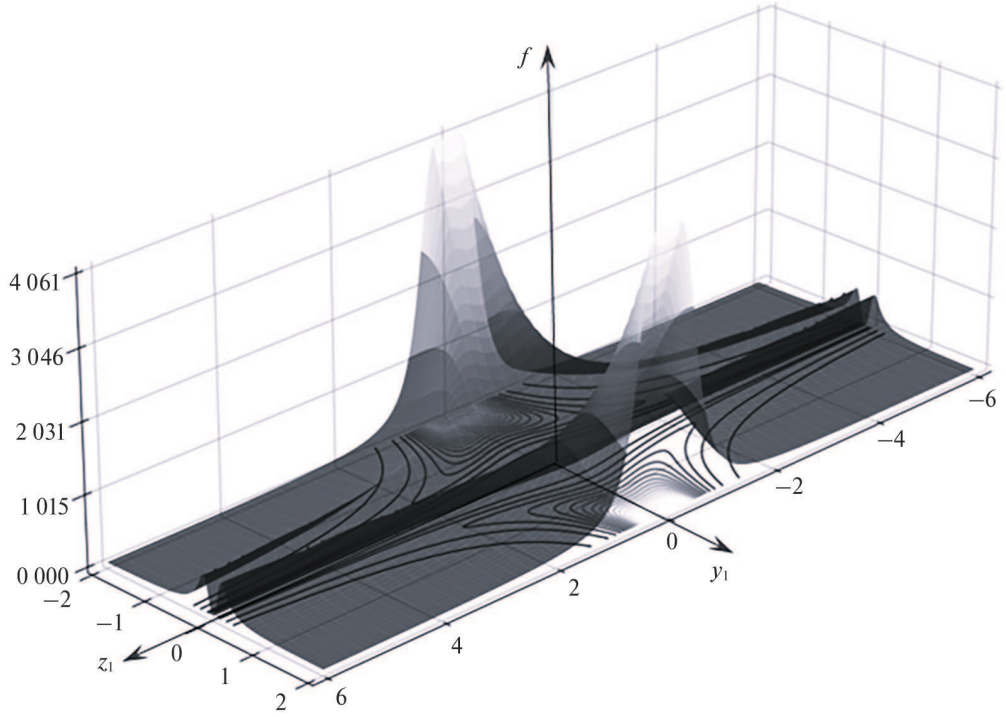


Fig. 1. Surface (13).

Under these assumptions, the energy integral $T + \Pi = \text{const}$ holds and, consequently, the pair $\{V(\mathbf{x}, \mathbf{x}') = T(\mathbf{x}, \mathbf{x}') + \Pi(\mathbf{x}),\ \boldsymbol{\mu} = \boldsymbol{\mu}(\mathbf{x})\}$ solves the partial stability problem of the equilibrium $\mathbf{x} = \mathbf{x}' = \mathbf{0}$: uniform stability with respect to $\mathbf{y}$ and $\mathbf{x}'$ (part of the generalized coordinates and all generalized velocities).

In particular, consider the motion of a unit-mass point on the surface

$$f = \frac{y_1^2(1 + z_1^2)}{1 + y_1^4 z_1^4} \tag{13}$$

above the horizontal plane Oy_1z_1 [4, 12].

In this case, $\mathbf{x} = (y_1, z_1)$,

$$T(\mathbf{x}, \mathbf{x}') = \frac{1}{2}\left\{y_1'^2 + z_1'^2 + \left[\frac{\partial f}{\partial y_1}y_1' + \frac{\partial f}{\partial z_1}z_1'\right]^2\right\}, \quad \text{and} \quad \Pi(\mathbf{x}) = gf(y_1, z_1),$$

where g denotes the acceleration of gravity.

We specify the pair as follows:

$$\{V(\mathbf{x}, \mathbf{x}') = T(\mathbf{x}, \mathbf{x}') + \Pi(\mathbf{x}),\ \mu_1(\mathbf{x}) = y_1z_1\}, \tag{14}$$

so the main Lyapunov function $V(\mathbf{x}, \mathbf{x}')$ has the structure (11) if $\mathbf{y} = (y_1, \mathbf{x}')$, and $W(\mathbf{x}, \mathbf{x}')$ is defined by the bracketed term in the expression for $T(\mathbf{x}, \mathbf{x}')$.

For sufficiently small h_1, we have

$$V(\mathbf{x}, \mathbf{x}') \geqslant a(|y_1| + |\mu_1| + \|\mathbf{x}'\|), \quad V'(\mathbf{x}, \mathbf{x}') \equiv 0$$

in the domain (12). Therefore, the equilibrium $\mathbf{x} = \mathbf{x}' = \mathbf{0}$ of the point is uniformly stable with respect to y_1 and $\mathbf{x}'$.

At the same time, for $\mathbf{y} = (y_1, \mathbf{x}')$ the condition $V(\mathbf{x}, \mathbf{x}') \geqslant a(|y_1| + \|\mathbf{x}'\|)$ fails in the domain (4): $V(\mathbf{x}, \mathbf{x}') \to 0$ as $|z_1| \to \infty$ for $\mathbf{x}' = \mathbf{0}$ and any fixed value of y_1. Hence, the Lyapunov function $V(\mathbf{x}, \mathbf{x}')$ in the pair (14) is indefinite with respect to y_1 and $\mathbf{x}'$ in the sense of [2] and does not satisfy the first condition of (5) in the domain (4).

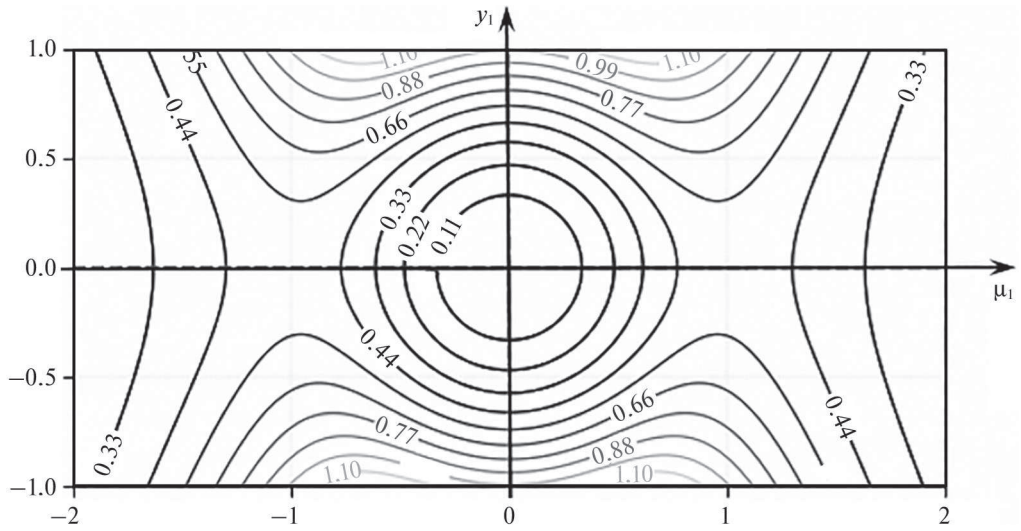


Fig. 2. Level lines of the surface (13) on the plane (y_1, μ_1).

The pair (14) serves to reveal a nonlocal nature of the uniform stability with respect to y_1 and $\mathbf{x}'$ and to estimate the domain of admissible initial perturbations $\mathbf{x}_0$ and $\mathbf{x}'_0$ of the variables $\mathbf{x}$ and $\mathbf{x}'$, respectively. Indeed, as the main function $V(y_1, z_1, \mathbf{x}')$ satisfies the condition $V(0, z_1, \mathbf{0}) \equiv 0$, the uniform stability of the equilibrium $\mathbf{x} = \mathbf{x}' = \mathbf{0}$ of the point with respect to y_1 and $\mathbf{x}'$ will hold not only for sufficiently small values of y_{10} and $\|\mathbf{x}'_0\|$ but also for large (finite) values of z_{10}.

As a supplement to [4, 12], it can be shown that for a sufficiently small value of $\|\mathbf{x}'_0\|$, the domain of admissible initial values y_{10} and z_{10} is given by the inequality

$$\frac{y_{10}^2(1+z_{10}^2)}{1+y_{10}^4 z_{10}^4} < \frac{1}{2}. \tag{15}$$

To prove this fact, we consider the equation $f(y_1, z_1) = c$ $(c = \text{const} > 0)$, defining the level lines of the surface (13). In terms of the variables (y_1, μ_1), this equation can be represented as a biquadratic one in μ_1 :

$$cr^2 - r + (c - y_1^2) = 0 \quad (r = \mu_1^2). \tag{16}$$

For $4cy_1^2 \geqslant 4c^2 - 1$, equation (16) has the two real solutions

$$r_1(c, y_1) = \frac{1 - \sqrt{1 - 4c(c - y_1^2)}}{2c}, \quad r_2(c, y_1) = \frac{1 + \sqrt{1 - 4c(c - y_1^2)}}{2c}.$$

In view of Vieta's formulas, we conclude that 1) $r_1 > 0$ and $r_2 > 0$ for $0 < c < 1/2$, $y_1^2 < c$, and also for $c \geqslant 1/2$ and $c - 1/(4c) \leqslant y_1^2 < c$; 2) $r_2 > 0$ for $y_1^2 > c$ and any $c > 0$. As a consequence, $r_2 > 0$ for $0 < c < 1/2$ and all y_1.

Let us analyze the solution $r_1 > 0$ in the case $0 < c < 1/2$. Since $r_1 \to 0$ as $c \to 0$, and $r_1 \to 1$ as $c \to 1/2$ and $y_1 \to 0$, we can identify the domain $y_1^2 < 1/2$, $\mu_1^2 < 1$ on the plane (y_1, μ_1) where the level lines of the surface (13) are closed elliptic curves encircling the point $y_1 = \mu_1 = 0$ and intersecting the line $y_1 = 0$ at the points $\mu_1 = \pm\sqrt{r_1}$.

In the domain $(y_1^2 < 1/2) \bigcap (y_1^2 < 1/z_1^2)$ on the plane (y_1, z_1), these level lines are associated with the open curves encircling the line $y_1 = 0$ and asymptotically approaching it as $c \to 0$ and $|z_1| \to \infty$.

The second branches of the level lines in the case $0 < c < 1/2$ (corresponding to the solution $r_2 > 0$), as well as the level lines for $c \geqslant 1/2$, are defined by the curves lying outside the indicated

domains. Indeed, $r_2 \to +\infty$ as $c \to 0$, and $r_2 \to 1$ as $c \to 1/2$ and $y_1 \to 0$, and the second branches of the level lines are open curves in the domain $\mu_1^2 \geqslant 1$ on the plane (y_1, μ_1) (intersecting the line $y_1 = 0$ at the points $\mu_1 = \pm\sqrt{r_2}$) and open curves in the domain $y_1^2 \geqslant 1/z_1^2$ on the plane (y_1, z_1). For $c \geqslant 1/2$, the level lines of the surface (13) lie in the domain $y_1^2 \geqslant 1/2$.

As a result, for a sufficiently small value of $\|\mathbf{x}_0'\|$ and values of y_{10} and z_{10} from the domain (15), it is possible to make the value $V(\mathbf{x}_0, \mathbf{x}_0')$ of the function $V(\mathbf{x}, \mathbf{x}')$ less than any arbitrarily small number and, furthermore, to place the point between the level lines of the surface (13) encircling the line $y_1 = 0$. Based on the identity $V'(\mathbf{x}, \mathbf{x}') \equiv 0$, we conclude that for all $t \geqslant t_0$, the point will not leave the indicated level lines of the surface (13).

Due to inequality (15), the uniform stability of the equilibrium $\mathbf{x} = \mathbf{x}' = \mathbf{0}$ of the point with respect to y_1 and $\mathbf{x}'$ is possible for sufficiently small values of z_{10} and $\|\mathbf{x}_0'\|$ and for $y_{10}^2 < 1/2$.

4. IDENTIFICATION OF SOME REFERENCE SYSTEMS

Using the proposed approach, we identify some structural forms of nonlinear systems of two and three equations for which the pairs under consideration can be constructed. Hereinafter, the functions $f_i(y_1)$ $(i = 1, 2)$ either have the form $d_i y_1$ with constants d_i or satisfy the conditions $|f_i(y_1)| \leqslant A|y_1|^{1+\alpha}$ with positive constants A and α in the domain $|y_1| < h_1$.

1. Let system (1) be composed of the equations

$$y_1' = ay_1 + ly_1^2 z_1, \quad z_1' = bz_1 + f_1(y_1)z_1 \tag{17}$$

with constant parameters a, b, and l.

We will establish conditions on the parameters under which the equilibrium $y_1 = z_1 = 0$ of system (17) is uniformly asymptotically y_1-stable.

By introducing the additional variable $\mu_1 = y_1 z_1$, one identifies the system

$$y_1' = ay_1 + ly_1\mu_1, \quad \mu_1' = (a+b)\mu_1 + l\mu_1^2 + \mu_1 f_1(y_1) \tag{18}$$

from the equations

$$y_1' = ay_1 + ly_1\mu_1, \quad \mu_1' = (a+b)\mu_1 + l\mu_1^2 + \mu_1 f_1(y_1), \quad z_1' = bz_1 + f_1(y_1)z_1.$$

If

$$a < 0, \quad a + b < 0, \tag{19}$$

then system (18) can be regarded as one of type (10) in the pair design scheme (Section 3) as applied to the original system (17): the equilibrium $y_1 = \mu_1 = 0$ of system (18) is asymptotically stable with respect to y_1 and μ_1 by Lyapunov's stability theorem based on the linear approximation.

The possibility of identifying system (18) and the admissible choice of $V = y_1^2 + \mu_1^2$ as a Lyapunov function for this system lead to the pair

$$\{V(\mathbf{x}) = y_1^2 + (y_1 z_1)^2,\ \mu_1(\mathbf{x}) = y_1 z_1\}. \tag{20}$$

The main Lyapunov function $V(\mathbf{x})$ in the pair (20) has the structural form (9), and under conditions (19), its derivative V' along the trajectories of system (17),

$$V' = 2\{ay_1^2 + (a+b)\mu_1^2(\mathbf{x}) + l\mu_1(\mathbf{x})[y_1^2 + \mu_1^2(\mathbf{x})] + \mu_1^2(\mathbf{x})f_1(y_1)\},$$

satisfies conditions (8) for $u_1 = [y_1, \mu_1(\mathbf{x})]$ in the domain (6), although it is *indefinite* in the domain (4).

As a result, under conditions (19) the equilibrium $y_1 = z_1 = 0$ of system (17) is uniformly asymptotically y_1-stable by Theorem 1. Moreover, the main function $V(y_1, z_1)$ in the pair (20) also satisfies the condition $V(0, z_1) \equiv 0$ and, consequently, uniform y_1-stability holds not only for sufficiently small values of y_{10} but also for large (finite) values of z_{10}.

The domain of attraction in system (18) can be estimated by known methods [22] using a Lyapunov function from a parametric class of quadratic forms in the variables y_1 and μ_1. In this case, on the plane (y_1, z_1) we obtain the corresponding domain of y_1-attraction of the equilibrium $y_1 = z_1 = 0$ of system (17).

2. Let system (1) be composed of the equations

$$\begin{gathered} y_1' = ay_1 + lz_1z_2, \\ z_1' = bz_1 + f_1(y_1)z_1, \quad z_2' = cz_2 + f_2(y_1)z_2 \end{gathered} \tag{21}$$

with constant parameters a, b, c, and l.

By introducing the variable $\mu_1 = z_1z_2$, one identifies the system

$$y_1' = ay_1 + l\mu_1, \quad \mu_1' = (b + c)\mu_1 + [f_1(y_1) + f_2(y_1)]\mu_1,$$

thereby predetermining the choice of the pair

$$\{V(\mathbf{x}) = y_1^2 + (z_1z_2)^2,\ \mu_1(\mathbf{x}) = z_1z_2\}.$$

Similar to case 1, we can demonstrate that the equilibrium $y_1 = z_1 = z_2 = 0$ of system (21) is uniformly asymptotically y_1-stable under the conditions $a < 0$ and $b + c < 0$. Moreover, uniform y_1-stability holds not only for sufficiently small values of y_{10} and z_{10} (z_{20}) but also for large (finite) values of z_{20} (z_{10}).

3. Let system (1) be composed of the equations

$$\begin{gathered} y_1' = ay_1 + ly_1^2z_1z_2, \\ z_1' = bz_1 + f_1(y_1)z_1, \quad z_2' = cz_2 + f_2(y_1)z_2. \end{gathered} \tag{22}$$

By introducing the variable $\mu_1 = y_1z_1z_2$, one identifies the system

$$y_1' = ay_1 + ly_1\mu_1, \quad \mu_1' = (a + b + c)\mu_1 + l\mu_1^2 + [f_1(y_1) + f_2(y_1)]\mu_1,$$

thereby predetermining the choice of the pair

$$\{V(\mathbf{x}) = y_1^2 + (y_1z_1z_2)^2,\ \mu_1(\mathbf{x}) = y_1z_1z_2\}.$$

Under the conditions $a < 0$ and $a + b + c < 0$, the equilibrium $y_1 = z_1 = z_2 = 0$ of system (22) is uniformly asymptotically y_1-stable. Moreover, uniform y_1-stability holds not only for sufficiently small values of y_{10} but also for large (finite) values of z_{10} and z_{20}.

4. Let system (1) be composed of the equations

$$\begin{gathered} y_1' = ay_1 + l_1y_1^2z_1 + l_2y_1z_1z_2, \\ z_1' = bz_1 + f_1(y_1)z_1, \quad z_2' = cz_2 + f_2(y_1)z_2. \end{gathered} \tag{23}$$

By introducing the variables $\mu_1 = y_1z_1$ and $\mu_2 = z_1z_2$, one identifies the system

$$\begin{gathered} y_1' = ay_1 + l_1y_1\mu_1 + l_2y_1\mu_2, \\ \mu_1' = (a + b)\mu_1 + l_1\mu_1^2 + \mu_1f_1(y_1) + l_2\mu_1\mu_2, \\ \mu_2' = (b + c)\mu_2 + [f_1(y_1) + f_2(y_1)]\mu_2, \end{gathered}$$

thereby predetermining the choice of the pair

$$\{V(\mathbf{x}) = y_1^2 + (y_1z_1)^2 + (z_1z_2)^2,\ \boldsymbol{\mu}(\mathbf{x}) = (y_1z_1, z_1z_2)\} \tag{24}$$

with a vector-valued auxiliary function $\boldsymbol{\mu}(\mathbf{x})$.

Under the conditions $a < 0$, $a + b < 0$, and $b + c < 0$, the equilibrium $y_1 = z_1 = z_2 = 0$ of system (23) is uniformly asymptotically y_1-stable. Moreover, uniform y_1-stability holds not only for sufficiently small values of y_{10} and z_{20} but also for large (finite) values of z_{10}.

5. Let system (1) be composed of the equations

$$\begin{gathered} y_1' = ay_1 + l_1y_1^2z_2 + l_2y_1z_1z_2, \\ z_1' = bz_1 + f_1(y_1)z_1, \quad z_2' = cz_2 + f_2(y_1)z_2. \end{gathered} \tag{25}$$

By introducing the variables $\mu_1 = y_1z_2$ and $\mu_2 = z_1z_2$, one identifies the system

$$\begin{gathered} y_1' = ay_1 + l_1y_1\mu_1 + l_2y_1\mu_2, \\ \mu_1' = (a + c)\mu_1 + l_1\mu_1^2 + \mu_1f_2(y_1) + l_2\mu_1\mu_2, \\ \mu_2' = (b + c)\mu_2 + [f_1(y_1) + f_2(y_1)]\mu_2. \end{gathered}$$

Under the conditions $a < 0$, $a + c < 0$, and $b + c < 0$, the equilibrium $y_1 = z_1 = z_2 = 0$ of system (25) is uniformly asymptotically y_1-stable. Moreover, uniform y_1-stability holds not only for sufficiently small values of y_{10} and z_{10} but also for large (finite) values of z_{20}.

The pair solving the problem is

$$\{V(\mathbf{x}) = y_1^2 + (y_1z_2)^2 + (z_1z_2)^2,\ \boldsymbol{\mu}(\mathbf{x}) = (y_1z_2, z_1z_2)\}.$$

5. USING REFERENCE SYSTEMS AS FIRST-APPROXIMATION SYSTEMS

The identified structural forms can be used as a nonlinear approximation for systems with polynomial nonlinearities.

1. Consider the perturbed system (17) composed of the equations

$$y_1' = ay_1 + ly_1^2z_1 + Y_1(t, \mathbf{x}), \quad z_1' = bz_1 + f_1(y_1)z_1 + Z_1(t, \mathbf{x}). \tag{26}$$

By introducing the variable $\mu_1 = y_1z_1$, the system can be transformed to

$$\begin{gathered} y_1' = ay_1 + ly_1\mu_1 + Y_1(t, \mathbf{x}), \quad \mu_1' = (a + b)\mu_1 + l\mu_1^2 + \mu_1f_1(y_1) + Y_1^*(t, \mathbf{x}), \\ z_1' = bz_1 + f_1(y_1)z_1 + Z_1(t, \mathbf{x}), \\ Y_1^*(t, \mathbf{x}) = y_1Z_1(t, \mathbf{x}) + z_1Y_1(t, \mathbf{x}). \end{gathered} \tag{27}$$

Using the analysis of system (17) and the pair (20), we can show that the equilibrium $y_1 = \mu_1 = z_1 = 0$ of system (27) is uniformly asymptotically (y_1, μ_1)-stable under conditions (19) and

$$|Y_1(t, \mathbf{x})| + |Y_1^*(t, \mathbf{x})| \leqslant \alpha_1|y_1| + \alpha_2|y_1z_1| \tag{28}$$

with sufficiently small positive constants α_1 and α_2 in the domain (6).

Therefore, under the above conditions, the equilibrium $y_1 = z_1 = 0$ of the original system (26) is uniformly asymptotically y_1-stable.

Condition (28) is valid, e.g., for $Y_1 = y_1^2z_1^2$, $Z_1 = y_1z_1^2$, and $Z_2 = y_1^2z_1$.

2. Consider the perturbed system (23) composed of the equations

$$\begin{gathered} y_1' = ay_1 + l_1 y_1^2 z_1 + l_2 y_1 z_1 z_2 + Y_1(t,\mathbf{x}), \\ z_1' = bz_1 + f_1(y_1)z_1 + Z_1(t,\mathbf{x}), \quad z_2' = cz_2 + f_2(y_1)z_2 + Z_2(t,\mathbf{x}). \end{gathered} \tag{29}$$

Using the analysis of system (23) and the pair (24), we can show that the equilibrium $y_1 = z_1 = 0$ of system (29) is uniformly asymptotically y_1-stable under the conditions $a < 0$, $a + b < 0$, $b + c < 0$, and

$$\begin{gathered} |Y_1(t,\mathbf{x})| + |Y_1^*(t,\mathbf{x})| + |Z_1^*(t,\mathbf{x})| \leqslant \alpha_1|y_1| + \alpha_2|y_1 z_1| + \alpha_3|z_1 z_2|, \\ Z_1^*(t,\mathbf{x}) = z_1 Z_2(t,\mathbf{x}) + z_2 Z_1(t,\mathbf{x}), \end{gathered} \tag{30}$$

with sufficiently small positive constants α_i in the domain (6).

Condition (30) is valid, e.g., in the case $Y_1 = y_1^2 z_1^2$, $Z_1 = y_1^2 z_1$, $Z_2 = y_1 z_1 z_2^2$.

6. CONCLUSIONS

This paper has considered systems of nonlinear ordinary differential equations with a continuous right-hand side. A possible approach has been proposed to find Lyapunov functions for the partial stability analysis of the trivial equilibrium of such systems (with respect to part of the variables). Within the approach, a main Lyapunov function–auxiliary function pair is chosen, where the latter function is used to correct the domain of the state space of the system where the former function is introduced.

A method has been presented to construct such pairs by transforming the original system and using partial stability conditions based on the first (linear or nonlinear) approximation.

In several illustrative examples, such pairs have been constructed for some nonlinear systems of two and three differential equations, with the main Lyapunov function defined as a quadratic form of the variables selected for stability analysis and additional variables. For these systems, partial stability domains have been found in the parameter space.

An analogous approach to partial stability analysis was considered for discrete-time systems [37] as well as for stochastic systems with continuous- and discrete-time dynamics [38–40].

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