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CONTROL IN TECHNICAL SYSTEMS

Optimal Navigation in Regular Transport Networks with Minor Traffic Congestion

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Abstract—Questions of the organization of movement in transport networks are considered. The emphasis is placed on the most simple and cheap solutions. Optimal nodal navigation in a network with insignificant traffic jams is sought. It is proved that, for a square grid, the fastest movement to the origin of coordinates will be obtained when the route tends to the coordinate diagonals with subsequent winding on them.

Keywords: flows on a graph, Hippodamian system, nodal navigation, insignificant traffic jams, control with incomplete information, time-optimal problem, cost of control

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1. INTRODUCTION

At present, when automobiles move in cities, the use of navigators has become practically universal. They solve several basic tasks at once: access to the city map, determination of the current location, obtaining in real time information about traffic jams and other characteristics of flows and their regulation, selection of the fastest route, and target indication to the driver in real time.

As a rule, such navigators are free for users, at least in their simplest tariffs, because of which questions of their cost are rarely raised. Nevertheless, substantial expenses constantly go to their functioning. These are street cameras, communication means, collection of information in real time, computing equipment, the staff of employees of various structures of traffic organization, taxes, etc. In other words, although final users do not see these costs, they exist and pass through the accounting departments of outside structures. And they are sometimes rather substantial in recalculation for each single movement. In such a situation the statement of the question of reducing such expenses is natural.

Although these problems are most clearly and sharply manifested in automobile flows of megacities, to one degree or another they are characteristic of all transport networks. In the most general case the question should be posed as follows: how to organize movement with the least accompanying costs when choosing a route and following it?

At present, questions of transport planning are so extensive and nontrivial that they have taken shape as a separate scientific discipline [1]. Although Dijkstra's algorithm for finding the shortest route in a graph is polynomial [2, 3], for large cities with millions of nodes this problem has become sufficiently cumbersome. Individual drivers without complex information-computing systems can no longer solve it well. As a result, flows become more and more chaotic and subject to the influence of outside information factors, and movements themselves around the city lose efficiency. Instead of choosing optimal solutions when forecasting flows, probabilistic approaches based on considerations of repetitions of past situations have more and more often begun to be used. At the same time, modeling of transport flows in real time is receiving ever greater spread in the world [4, 5].

The hopes of recent years for the use of modern neural networks, even embedded in basic mechanisms of deductive reasoning [6, 7], also can only partially help to cope with the complexity. Say, for them examples of features of architectural building are needed, which are not explicitly present in the initial data used for training the neural network.

All this involuntarily pushes to the thought that the problem is excessively complicated [8]. And it is necessary to try to simplify it as much as possible. As often happens, most transport problems are conditioned by nontransport reasons. For example, for cities traffic jams, as a rule, are conditioned by wrong architectural and planning decisions, which accumulate for centuries, when for free movement of the flow there is simply not enough corresponding volume of roads [9]. Correspondingly, in these conditions many transport solutions in fact are also nontransport. As an example, one can point to intercepting parking lots or fiscal restrictions on owners of transport means with the aim of reducing their number in cities.

One of the main approaches for organizing qualitative movements over a territory is the formation of corresponding urban development on the planning horizon of tens of years. And first of all, the creation of the correct topology of the street-road network. It can be shown that in many cases it is expedient to choose regular networks, i.e., networks with a repeating structure, as target ones [10]. They are optimal by many parameters, including nontransport ones. Their value is confirmed by the fact that they are widely used in practice from ancient times.

In this article the simplest square network will be considered. Although it is not the most effective among regular ones [11], it is the simplest by structure and therefore the most convenient for building. It can be considered as a target one for many territories, allowing one to achieve a high quality of movements.

As in any other network, traffic jams can also arise in a square network. Therefore, for it the task of movements under conditions of incomplete information is also actual. In the article a very simple variant of navigation through a square network without the use of navigators or any other devices is proposed. It is proved that it leads to the fastest movement.

2. PROBLEM STATEMENT

Consider some finite weighted graph $G(V, E, L)$, where $V = \{v_i\}$ is the set of its vertices; $E = \{e_i\}$ is the set of edges connecting its vertices; $L = \{l_i\}$, $l_i > 0$, are positive lengths of edges [2]. We shall consider that this graph describes some transport network: the street-road network of a megacity, a railway network, or even beds in a field. In the general case, the task is posed of finding such rules of choosing movement from some initial vertex $u \in V$ in order finally to get to some terminal vertex $v \in V$.

Consider at first the simplest square regular network, which consists of vertical and horizontal roads. We choose, without loss of generality, such units of length that the coordinates of the vertices of the graph describing it would have integer values, and the length of the edges between vertices would be equal to one. Then the roads are described by the lines:

$$x_i = \pm n, \quad y_j = \pm m, \quad n, m \in \mathbb{Z}. \tag{1}$$

The distances between roads are also equal to one.

Such a network has great practical significance in the design of real transport systems. For example, when it is chosen, classical city blocks arise. Such a network has even received in urbanistics a special name, Hippodamian, after the name of the Greek architect Hippodamus, who applied it in the restoration of Miletus destroyed in the course of the Greco-Persian wars [12]. In the design of transport corridors on the scale of regions and whole continents, the latitudinal-meridional principle is often used, which also leads to a square network. And even beds in fields appear as a variety of this approach.

One can move along the roads of the network. We choose such a unit of time that the speed of movement would be equal to one. Then the time of movement between adjacent nodes will also be unit.

Without loss of generality, we place the initial point of departure of the route $u(n, m)$ in the first coordinate octant, i.e., with positive coordinates, $n > 0$, $m > 0$. And in the origin of coordinates $v(0, 0)$ we place the terminal point of the route. For such a network it is required to determine rules of choosing the direction of movement in each node in order to get by the shortest way from some point $u \in V$ to the origin of coordinates. This, for example, can be done if in each node such an edge is chosen, movement along which leads to a decrease of the distance to the origin of coordinates [13].

Let us note several simplest properties of this network.

Statement 1. *In a Hippodamian network, for any pair of points not lying on one straight line of the network, there exist several shortest routes of equal length.*

Corollary 1. *Under violations of passage along any one section of the network, for vertices not lying on one straight line, its bypass with the same route length will be found.*

Thus, for the Hippodamian network the property of stability of traffic length under violations is characteristic, which turns out to be very important for practical applications.

Definition 1. The distance or length $l_{n,m} = n + m$ of the network node (n, m) will be called the extent of its shortest route to the origin of coordinates of the node in a square network.

3. THE CASE OF TRAFFIC JAMS WITH SMALL PROBABILITY

Let us now complicate the task. Let on some edges of the graph there arise obstacles of some kind to free movement, which further we shall call traffic jams. The locations of traffic jams are not known in advance. I.e., initially this is a task of control under conditions of incomplete information. But, coming to any node, we receive additional information both about the node itself and about traffic jams on edges adjacent to it: for example, simply by looking at the corresponding road; or at a traffic light, besides the color permitting or prohibiting passage, some number of the same color can be shown, which will characterize the intensity of the traffic jam; or in some other way. In other words, for the driver this is local information, accessible only at the point of presence.

In such a statement of the task, the information used for choosing the further direction of movement, as well as for orientation and position in the network, is much more simple and therefore cheaper than information about the whole graph G. At the same time, such sufficiently expensive tasks are automatically excluded as collecting information about the structure of the graph and the current location of traffic jams; the use of an onboard computing device is not required, nor providing it with communication with the information supplier; it is not necessary to determine the optimal route (in the case of a large graph with millions of vertices this is a separate long task) and to make sure of its existence, to organize following the route. From this point of view such control of movement is much more simple and cheap. Further it will be shown that under conditions of such scanty information such control exists. And it will even lead to optimal movements. But this becomes possible only due to the very simple and repeating structure of a regular network [14].

We shall suppose that for the event that a traffic jam will be found on an edge, there exists some probability p; further we shall consider it a small quantity.

In practice, traffic jams can be very diverse in their properties. In this work we shall consider that a traffic jam is always passable, i.e., having begun movement along an edge with a traffic jam, we shall necessarily get to an adjacent node. But the time of passing the edge in this case increases by some quantity $d < 1$.

Statement 2. *If there is no information about the presence of a traffic jam on an edge, then the mathematical expectation of the time of passing this edge will be equal to* $1 + pd$.

Proof. There are two variants. Either there will be a traffic jam on this road, or there will not. If there will be a traffic jam on it, then the time of movement will be $1+d$; if there is no traffic jam, then it is 1. The probabilities of these events are respectively p and $1-p$. Then the mathematical expectation of the time $M(e)$ of passing the edge will be equal to:

$$M(e) = p(1+d) + (1-p) \cdot 1 = 1 + pd. \tag{2}$$

Corollary 2. *The expected average time of passing an edge will exceed the time of passing a free edge by a quantity not more than of the first order of smallness.*

As was said earlier, by the condition of the task it is not known in advance where and when traffic jams will arise. In such a statement of the task one cannot bypass traffic jams in advance, since information (not all) about the state of the network comes only in the course of movement. Correspondingly, the task of choosing the further direction of movement is also forced to be solved in the course of movement.

Definition 2. Movement from point to point, when from time to time the direction of movement is chosen, will be called navigational.

A distinctive feature of navigation is that the route is not known in advance, even when movement has already begun, even already having begun movement [15, 16]. This will depend on information about traffic jams coming during movement, i.e., the task is a variety of adaptive control.

If the choice of direction occurs only in nodes of the network, then such navigation is called nodal. Nodal navigation is characteristic of many real networks: railway and automobile roads, routers in computer networks, electric networks, gas and oil pipelines, etc. It has great practical significance, which is why it makes sense to distinguish it by a separate name.

Definition 3. We shall call any algorithm of choosing in each node the next edge for movement the choice rules π.

In this article the choice rules do not necessarily have to be unambiguous. In the case of probabilistic rules, they will determine the probabilistic dynamics of flows on the network, as in Markov networks.

4. TIME-OPTIMAL PROBLEM

For movement under conditions of traffic jams the former quality criterion, the minimum length of a route, becomes not actual. Now one should consider the time-optimal problem. The task is posed to find such rules π of route choice in order to reach the origin of coordinates in minimum time. Further only such rules π will be considered which always lead to the terminal point of the route.

Definition 4. We shall denote by the expression $t_\pi(u) = t_\pi(n,m)$ the mathematical expectation of the time of reaching the origin of coordinates from the point $u(n,m)$ under the choice rules π. The index π will be omitted further if it is implied by the context. If the rules π are optimal, then $t(u) = t(n,m) = t_{n,m}$ is the mathematical expectation of the least possible time.

The least quantity $t_{n,m}$ will be called the time of the node, and differences of quantities of the type $t_{n,m} - t_{n-1,m}$ or $t_{n,m} - t_{n,m-1}$ will be called the times of the corresponding edges for movement from the point (n, m).

By the condition of the task, the occurrence of traffic jams on edges does not depend on each other in any way, which is really observed under rare traffic jams. Therefore, in accordance with the principle of dynamic programming, the very notion of the time of a node is correct.

In what follows the following will be very useful.

Lemma 1. *For each sufficiently small p around the origin of coordinates there exists a domain σ, in which for each node of the network the time is equal to $t_{n,m} = n + m + O(p) = l_{n,m} + O(p)$.*

Proof. The proof follows from the smallness of the quantity p and Statement 2.

In other words, the metric in a neighborhood of the origin of coordinates will differ insignificantly from the initial Hippodamian one for a square network. Correspondingly, the smaller p is, the larger the domain σ will be. For any two points u_1 and v_1 the following will be true.

Corollary 3. *In the domain σ, if $l(u) < l(v)$, then $t(u) < t(v)$.*

Further we shall consider only nodes of the network which are situated to the right and below the diagonal of the first octant, including its boundaries, i.e., nodes $u(n, m)$ whose coordinates will satisfy the relations $0 \leqslant m \leqslant n$. For nodes from the upper diagonal half the reasoning will be analogous.

Corollary 4. *If the penalty $d < 1$, then in the domain σ the fastest routes can leave nodes only along the left and lower edges.*

Proof. When moving upward or to the right, the length of the node increases by one. For returning to the same level of distances it is necessary to spend at least one more unit of length. Since in the domain σ the hierarchy of distances to the origin of coordinates is preserved, passing through a traffic jam will be faster than its possible bypass.

Let us note that, for preserving most conclusions, the restriction $d < 1$ from above by one can be considerably improved to quantities close to 2. But this will strongly complicate the exposition without obtaining new qualitative advantages. It will be necessary to investigate the sizes of the domain σ, the form of its boundaries, after which the formulations of the statements will become much more cumbersome.

Corollary 4 makes it possible to find optimal control in the form of rules for choosing directions of movement in nodes. Consider at first two special cases of nodes on the abscissa axis and on the diagonal of the first coordinate octant.

Example 1. Let us find the values $t_{n,0}$. Consider at first the node $(1, 0)$ on the abscissa axis to the right of the origin of coordinates. In view of the symmetry of the task with respect to the coordinate axes and their diagonals, analogous reasoning can be given for the points $(0, 1)$, $(0, -1)$, $(-1, 0)$. Only movement along the road to the left brings closer to the origin of coordinates, since movement downward increases the distance $l(u)$. In accordance with the statement above, the mathematical expectation of the time of movement along it is equal to $t(1, 0) = 1 + pd$.

For other points on the abscissa axis of the type $(n, 0)$, for optimal control it is required analogously to choose only the direction to the left in view of the nonoptimality of bypassing traffic jams in the domain σ. Then $t_{n,0} = n(1 + pd)$.

Corollary 5. *The times of edges on the coordinate axes are greater than the lengths of edges by a quantity of the first order of smallness.*

Example 2. Let us find the values of the time of the node $t_{1,1}$ on the diagonal of the coordinate octant. In accordance with the principle of dynamic programming, for finding the least time of movement from each point one should choose that direction which will give the best result taking into account already known optimal routes.

By Corollary 4, among the directions of further movement one should choose either along the edge downward or along the edge to the left. From Example 1 it is known that the least mathematical expectations of times for the points $(1, 0)$ and $(0, 1)$ are equal and are equal to $1 + pd$. At the point $(1, 1)$ the following four variants are possible. Both necessary directions will be with traffic jams. The lower edge will be with a traffic jam, and the left one free. On the left there will be a traffic jam, the lower one free. Both edges will be free from traffic jams. Correspondingly, the probabilities of these events will be equal to p^2, $p(1 - p)$, $(1 - p)p$, $(1 - p)^2$.

Let us formulate the rules of choosing the direction π, consisting in that we shall choose an edge without traffic jams always when this is possible. If both edges are free or both are with traffic jams, then we choose any one with equal probability. Under such a choice, in three of four cases a free edge will be chosen. Then the mathematical expectation of the time of the diagonal node will be equal to:

$$t_{1,1} = p^2(1 + d + t_{1,0}) + p(1 - p)(1 + t_{1,0}) + (1 - p)p(1 + t_{0,1}) + (1 - p)^2(1 + t_{0,1}). \tag{3}$$

Since $t_{1,0} = t_{0,1} = 1 + pd$, the expression is simplified to:

$$t_{1,1} = (1 + pd) + 1 + p^2d = 2 + pd + p^2d. \tag{4}$$

Since $t_{1,0} = 1 + pd$, the time of the vertical edge $t_{1,1} - t_{1,0}$ is equal to $1 + p^2d$. And since the probability p is a small quantity, the times of the lower $t_{1,1} - t_{1,0}$ and left $t_{1,1} - t_{0,1}$ edges from the node $(1, 1)$ exceed their lengths by quantities not of the first, but already of the second order of smallness. In other words, they are faster than edges on the coordinate axes. Further it will be shown that a similar property can be ensured for all points outside the coordinate axes.

5. OPTIMAL NAVIGATION

Consider now the general case of an arbitrary node $u(n, m)$, whose coordinates will satisfy the relations $0 < m \leqslant n$, i.e., located in the half of the octant without the abscissa axis.

Definition 5. The rules of choosing the direction of movement π will be called diagonal if:

1) The choice is made only between the lower and left edges;
2) If the left edge is free, then it is chosen;
3) If there is a traffic jam on the left edge, and the lower one is free, then the lower one is chosen;
4) If there are traffic jams on both edges, then we choose the left one.

In other words, in the simultaneous presence of a pair consisting of a free edge and an edge with a traffic jam, the free edge is always chosen. In other cases the advantage always has the left edge. Under such rules the movement tends in the direction of the diagonal of the octant, whence its name was taken.

If in all nodes the diagonal choice rules are used, then in accordance with the principle of dynamic programming the times of nodes will be uniquely determined around the origin of coordinates. Consider some node (n, m). The time of the node to the left of it $(n-1, m)$, denote by $t_{left} = t_{n-1,m}$, and the time of the lower node $(n, m - 1)$ as $t_{down} = t_{n,m-1}$.

Statement 3. *For the diagonal rules π the recurrent relation is true*

$$t_{n,m} = t_{left} + 1 + (p - p^2)(t_{down} - t_{left}) + p^2d. \tag{5}$$

Proof. The mathematical expectation of time for the diagonal rules is collected from the probabilities of all possible states of the edges:

$$t_{n,m} = p^2(t_{left} + 1 + d) + p(1-p)(t_{left} + 1) + (1-p)p(t_{down} + 1) + (1-p)^2(t_{left} + 1). \quad (6)$$

Opening the brackets and grouping the summands, we obtain the required expression.

According to Lemma 1, the times of the nodes t_{left} and t_{down} differ from their lengths by quantities of the first order of smallness. But the lengths of these nodes are equal. Therefore the difference of the times t_{down} and t_{left} will also be of the first order of smallness. And the summand $(p-p^2)(t_{down} - t_{left})$ will already be of the second order of smallness. Therefore the time of the left edge differs from unity by a quantity of the second order of smallness.

Recall that on the abscissa axis the times of edges were greater than lengths by quantities of the first order of smallness.

Corollary 6. *For the zero and first horizontal layers of nodes of the network it is true that* $t_{n-1,1} < t_{n,0}$.

Statement 4. *If* $t_{n-1,m} < t_{n,m-1}$, *then the diagonal rules* π *are optimal control of movement of the time-optimal problem in the node* (n, m), *where* $0 < m \leqslant n$.

Proof. For the node $(1, 1)$ above the optimal control and its time were directly found. For the node $(1, 1)$ it is directly checked that the diagonal choice rules also give optimal control. Consider an arbitrary node (n, m). The proof is based on a direct check of each combination of traffic jams on the lower and left edges. If both edges are free, then the left edge is chosen, since the time of the left vertex is less than that of the lower one, and the times of the edges are equal and equal to unity. If the left edge is free, and the lower one is with a traffic jam, then both the time of the left edge is less and the time of the left node is less. If the left one is with a traffic jam, and the lower one is free, then the lower one is chosen, since by the conditions of the task the value of the penalty d when passing along the left edge will be greater than the difference of the times of the left and lower nodes, since in the domain σ this difference has the first order of smallness. If there are traffic jams on both edges, then we choose the left one, since the times of the edges are equal, and the time of the left node is less.

Statement 4 is also true in the opposite direction.

Lemma 2. *Under the diagonal choice rules* π *in the domain* σ *the inequality* $t_{n-1,m} < t_{n,m-1}$ *will be fulfilled.*

Proof. We shall carry out the proof by induction. The base of induction. Consider the nodes $(2, 2)$ and $(3, 1)$. For them it is true:

$$t_{2,2} = (1-p)^2(1 + t_{1,2}) + (1-p)p(1 + t_{1,2}) + p(1-p)(1 + t_{2,1}) + p^2(1 + d + t_{2,1}), \quad (7)$$

$$t_{3,1} = (1-p)^2(1 + t_{2,1}) + (1-p)p(1 + t_{2,1}) + p(1-p)(1 + t_{3,0}) + p^2(1 + d + t_{2,1}). \quad (8)$$

As for nodes symmetric with respect to the diagonal, in view of the symmetry of the task it will be true that $t_{1,2} = t_{2,1}$. And $t_{3,1} < t_{4,0}$ by Corollary 6, since the latter node lies on the abscissa axis. Comparing the expressions by each summand, we obtain that $t_{2,2} < t_{3,1}$. Thus we obtain three consecutive nodes along an oblique diagonal with times $t_{2,2}$, $t_{3,1}$, $t_{4,0}$ in the order of nondecrease and growth for the last pair. Further we shall call such triples of nodes oblique. There are also two horizontal layers, where the times of nodes of the upper layer along an oblique diagonal are less than the times of nodes of the lower layer.

The induction step. Let for some triple of nodes it be true that $t_{n-2,m} \leqslant t_{n-1,m-1} < t_{n,m-2}$, and also for any positive i the inequality $t_{n-1+i,m-1} < t_{n+i,m-2}$ is fulfilled. Consider two nodes

$(n-1,m)$ and $(n,m-1)$ to the right and above this triple. Their times are determined by the recurrent formulas

$$\begin{aligned} t_{n-1,m} = (1-p)^2(1+t_{n-2,m}) + (p-p^2)(1+t_{n-2,m}) \\ + (p-p^2)(1+t_{n-1,m-1}) + p^2(1+d+t_{n-2,m}), \end{aligned} \tag{9}$$

$$\begin{aligned} t_{n,m-1} = (1-p)^2(1+t_{n-1,m-1}) + (p-p^2)(1+t_{n-1,m-1}) \\ + (p-p^2)(1+t_{n,m-2}) + p^2(1+d+t_{n-1,m-1}). \end{aligned} \tag{10}$$

Each of the summands of the upper expression will be greater than or equal to the analogous summand of the lower formula. Moreover, the equality sign can be only for nodes of the type $(k-1,k)$ and $(k,k-1)$ around the diagonal of the octant. Substituting these inequalities, we obtain the statement being proved. Therefore, if there is an oblique triple, then to the right of it there will also be an oblique triple. I.e., they are arranged in layers horizontally. On the other hand, if there are not less than two horizontal layers $y=k$ and $y=k-1$ where $t_{i,k} < t_{i+1,k-1}$, then there will always be found an oblique triple above them. This will be a triple of nodes of the type $(k+1,k+1)$, $(k+2,k)$, $(k+3,k-1)$, i.e., beginning from the coordinate diagonal. The proof is analogous by the formulas above, only it is necessary to take into account that in view of the symmetry of the task there is the strict equality $t_{k,k+1} = t_{k+1,k}$.

Theorem 1. *The diagonal choice rules are optimal control of movement for the domain σ without the coordinate axes.*

As is seen from the previous exposition, the times of all edges, except those lying on the coordinate axes, exceed their lengths by a quantity not more than quantities of the second order of smallness. In other words, for most cases the diagonal rules provide a very high quality of control, practically eliminating the influence of traffic jams during movement.

The very principle of the striving of a route to the domain where there exists the greatest number of variants of continuation of movement and, correspondingly, of bypassing traffic jams, has a fundamental character. Therefore it is very plausible that it can be used under much less severe restrictions. Consider some directions for generalizing the results.

6. FINITE VALUES OF THE PROBABILITY p

Although formally the probability of occurrence of a traffic jam in the given statement of the task is considered an infinitely small quantity, for practical applications cases with finite values are of interest. In particular, the question is of interest how far the domain σ can extend. Here the following will be useful.

Statement 5. *The time of any edge is more than* 1 *and not more than* $1+pd$.

This follows from Statement 2 with the remark that, when approaching any node, additional information becomes accessible, which can reduce the mathematical expectation of the time of movement along it.

According to Corollary 3, in the domain σ, if $l(u) < l(v)$, then $t(u) < t(v)$. This can be considered as an alternative definition of the domain of applicability of the diagonal choice rules. The very notion of the domain σ exists by definition only under the limiting transition with respect to the probability p, which does not allow one to calculate its boundaries at some finite p. On the contrary, the given property is sufficiently easy to check. This makes it possible to obtain quantitative estimates of the size and form of the domain where the proposed control of movement will be optimal.

One of the rough estimates of the extent of the domain σ can be the condition $t(u) < l(u)+1$, which makes bypasses ineffective. In accordance with Statement 5, for $d<1$ the domain will include all nodes with $l(u) < 1/p$. For example, when the probability of a traffic jam is equal to one hundredth, the diameter of such a domain in a city will be of the order of 200 blocks. Few megacities in the world are of such an extent. In other words, this is although a rough estimate, but already having practical sense.

At high values of the probability p another difficulty arises. A chain reaction of generation of other traffic jams by traffic jams can begin because of the reduction of the throughput capacity of the whole network. Then the simultaneous appearance of a traffic jam on the left and lower edges can begin to arise successively during movement, which will require taking into account not only the nearest edges, but also more distant ones. And this requires both more extensive information about the current state of the graph and a modification of the choice rules themselves. I.e., the very statement of the task changes. How strongly the parameter p can be increased remains an open question.

7. PENALTY FOR PASSING A TRAFFIC JAM

In the current statement of the task it was supposed that a traffic jam is always passable. And the undesirability of its passing was given by the quantity of the time penalty d, which was chosen less than unity. This was done in order that the bypass would be nonoptimal. Indeed, the diagonal rules lead to routes with a successive decrease of the lengths of the passed nodes. In them the principle of bypassing traffic jams on the horizon of observability is already laid. And any bypass outside diagonal directions leads to an overrun in length by 2 units.

Variants of statements of the task with large values of d are also possible. And they really arise in practice, which conditions interest in them. But under such quantities bypasses of traffic jams become optimal. And this requires additional assumptions to the initial statement of the task at the methodological level.

When choosing a bypass, it is extremely desirable to expand the zone of informing drivers about more distant edges, and not only adjacent ones. And this contradicts the aims of this article, where, on the contrary, the emphasis is placed on simplifying movement and reducing the cost of its accompaniment. At the same time, one can also inform in different ways. For example, one can give information about the state of edges removed from the current position through one vertex. Or one can give the whole current graph of the network. Depending on the chosen variant, a whole set of independent statements of tasks with their own features and domain of application arises.

It is necessary to distinguish the types of traffic jams. For example, they can be constant, and they can change after some interval of time. In the latter case it is necessary explicitly to indicate the law of occurrence and disappearance of congestions. This leads to new series of mathematical models. A separate case arises when movement through an edge is prohibited, for example because of the closure of a road section for repair or because of some other insurmountable reason. In this case there may generally not exist any route to the origin of coordinates, i.e., the task becomes incorrect.

Despite the indicated and other difficulties, cases of bypasses of traffic jams are very important for practice. But at present there does not exist a unified approach to solving such a task. In cities, navigators with display of the location of traffic jams and their intensities are most often used. At this it is necessary to remember that such information comes with a delay. Sometimes explicit indicators of bypasses help. In traffic control centers, expensive methods of modeling and forecasting are very often used. At the same time, probabilistic approaches and expert estimates based on knowledge about past analogous situations are used.

In the most neglected cases high values of p and d will lead to situations when in some parts of the network movement will generally stop. Then there arises already the task not of choosing movement, but generally of the existence of at least one possible route. For example, tasks of flow in a percolation cluster arise [17], which are very complex even for theoretical investigation.

The absence of a good cheap solution says that road networks with high values of the probability of occurrence of a traffic jam and high costs of penalties must be avoided in every possible way. Including the use of nontransport methods: architectural and planning decisions, traffic restrictions, stimulation of small sizes of automobiles, reduction of the number of automobile owners, compact cities, etc.

8. CONCLUSION

The diagonal rules generate optimal nodal navigation when moving in a square network with small traffic jams in some domain around the origin of coordinates.

The diagonal rules admit a very simple and convenient for practical use interpretation. It consists in the striving to the diagonals of the coordinate octant. Upon reaching the diagonal, the winding of the route onto the diagonal begins. This makes it possible to move through the network in an optimal way without the use of any additional computing or information devices. This becomes possible because under such choice rules the greatest number of variants of bypassing traffic jams is achieved.

The proposed control can be chosen as a target one when constructing real transport networks with regular topology in view of its simplicity, stability of traffic length, and cheapness. Such an approach is an alternative to the existing trends toward complication of the mechanisms of control and management of real transport flows, and correspondingly of the growth of costs for them.

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